Centre of Mass (or Centre of Gravity)

Look at the toys
Man on horse, leaning tower of Pisa, rolling doweling.

Why doesn't the horseguy fall over?
Why doesn't the leaning tower fall over?
Why does the wooden double cone thing roll up the hill?

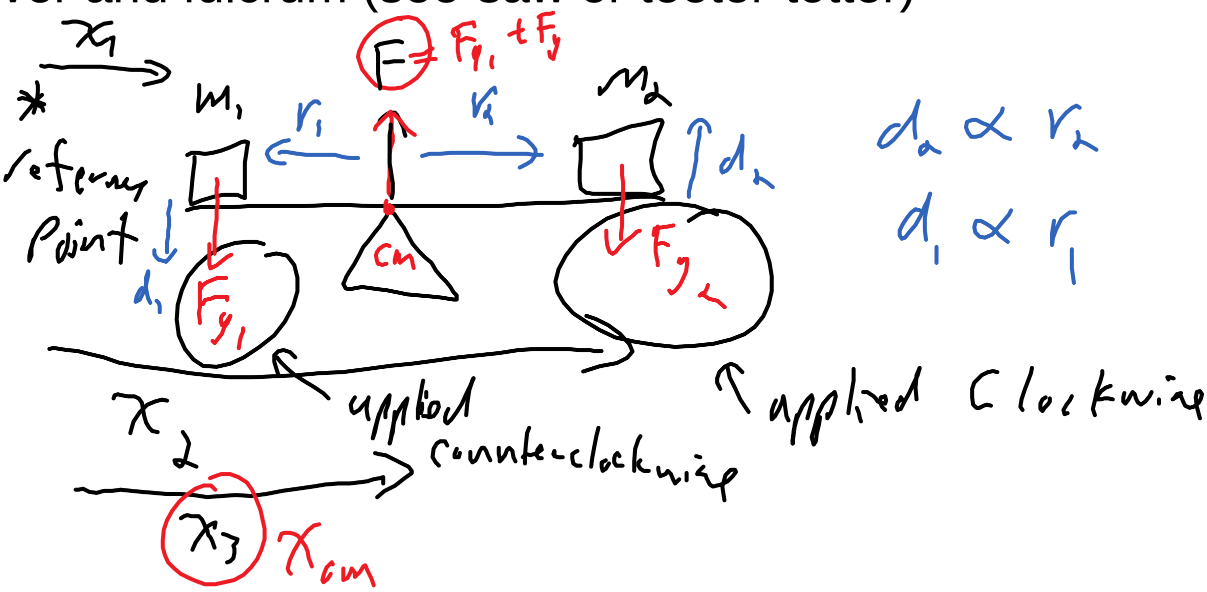
Draw each apparatus and write a little description

<u>Centre of Mass and Statics</u>

Centre of Mass (cm) or centre of gravity (cg)
NOT where you have half the mass on one side and half on the other. WRONG

We can derive our equation by conservation of energy and knowing it is related to the distribution of mass - determined by the position as well as the mass.

Look at a lever and fulcrum (see-saw or teeter-totter)



if energy is conserved $\Delta E_{nergy} = 0$

$W_1 + W_2 = 0$

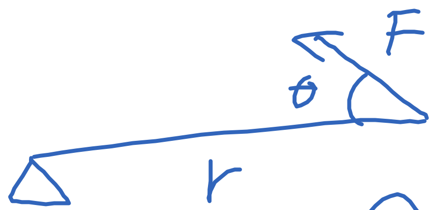
$w_1 + w_2 = 0$

$W_{F_{g1}} = W_{F_{g2}}$

$F_{y1} d_1 = -F_{y2} d_2$

$F_{g1} r_1 = -F_{g2} r_2$ ← call this torque, τ

Torque is the vector cross product of force and radial distance.



$\tau = F r \sin\theta$

↑ greek letter Tau

↑ ($\sin$) max when $F \perp r$

$\tau = F r \sin\theta$

Translational equilibrium - you are not accelerating - sum of all forces = 0

$F_{up}=-F_{down}$ $F_{right}=-F_{left}$

Rotational equilibrium - you are not spinning faster or slower - sum of all torques = 0

Torques clockwise = -torques counter clockwise

Static equilibrium - both rotational and translational equilibrium.

Let's go back to the centre of mass of m_1 and m_2.

$$\tau_c = \tau_{cc}$$

clockwise   counter clockwise

$$F_{g_2} r_2 = F_{g_1} r_1$$

$$m_2 g r_2 = m_1 g r_1$$

$$m_2 r_2 = m_1 r_1$$

Set reference point outside

$$\tau_c = \tau_{cc}$$

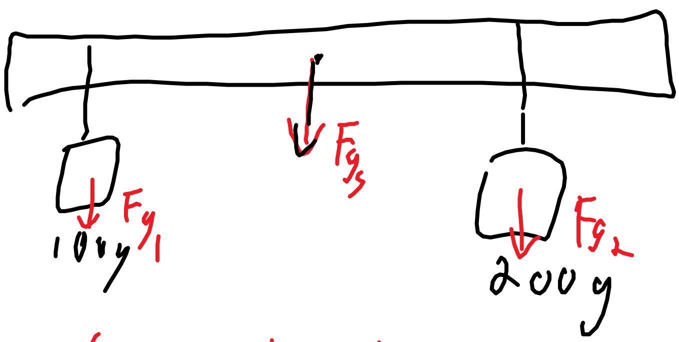
$$F_{g_1} x_1 + F_{g_2} x_2 = (F_{g_1} + F_{g_2}) x_{cm}$$

$$m_1 g x_1 + m_2 g x_2 = (m_1 g + m_2 g) x_{cm}$$

$$m_1 x_1 + m_2 x = (m_1 + m_1) x_{cm}$$

Eg. A 100g mass is connected to a 150 g metre stick at 10 cm and a 200g mass is connected at 80 cm. Where does the system balance (centre of mass)? What force do I have to push up on the stick?

Trick → Set the mass of metre stick at 50cm - centre.

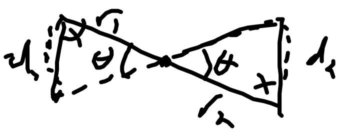


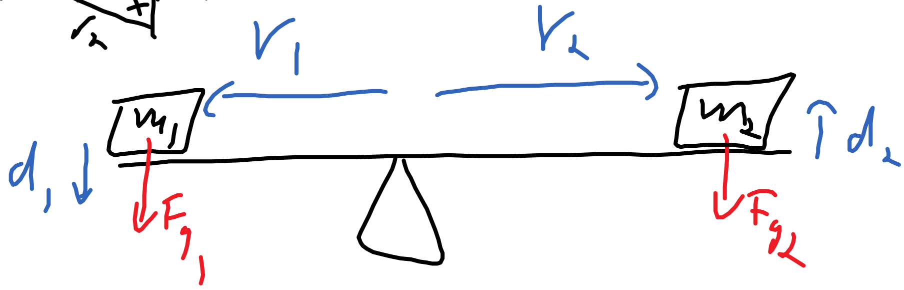
Set ref point at 0

$$m_1 x_1 + m_2 x_2 + m_3 x_3 = (m_1 + m_2 + m_3) x_{cm}$$

$$100g(10cm) + 200g(80cm) + 150g(50cm) = (100g + 200g + 150g) x_{cm}$$

$$x_{cm} = 54 cm$$





fulcrum
(lever rotation point)

by conservation of energy

$$W_1 = W_2$$

$$F_{g_1} d_1 = F_{g_2} d_2$$

$$m_1 g d_1 = m_2 g d_2$$

$d_1 \propto r_1$

$d_2 \propto r_2$

$$\frac{m_1}{m_2} = \frac{d_2}{d_1} = \frac{r_2}{r_1}$$

$$\boxed{m_1 r_1 = m_2 r_2}$$

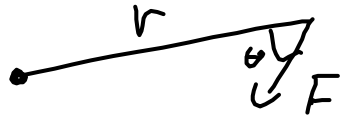
r is the radial distance perpendicular to the force

Torque, $\tau = F \times r$

↑ greek letter Tau

Torque, τ, is the vector cross product of force, F, and radial distance, r.
It is max when F is perpendicular to r.

$\tau = Fr\sin\theta$



Units: Nm (don't use Joules)

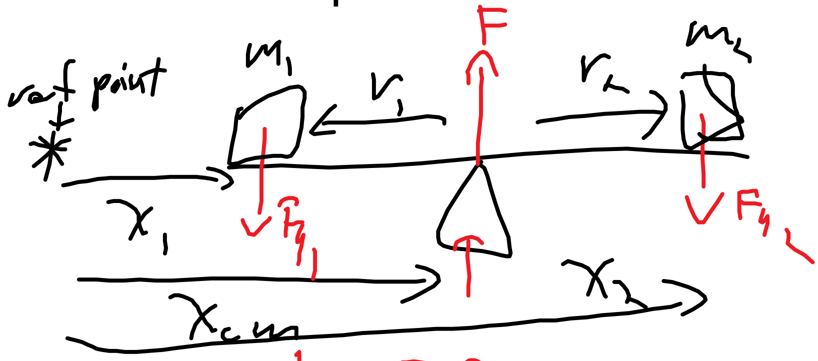
Translational equilibrium - you don't accelerate
F is balanced $F_{net}=0$
$F_{up}=F_{down}$ $F_{right}=F_{left}$

Rotational equilibrium - you don't rotate faster of slower.

Torques are balanced $\Sigma\tau=0$

Clockwise Torques = counter clockwise torques

Statics - when you have both translational and rotational equilibrium



clockwise = counter-clockwise

$\tau_c = \tau_{cc}$

$F_{g_2} r_2 = F_{g_1} r_1$

what's F? $F = F_{g_1} + F_{g_2}$

repeat with a difference reference point

$\tau_c = \tau_{cc}$

Now both F_{g_1} and F_{g_2} are clockwise

$$F_{g_1} x_1 + F_{g_2} x_2 = F x_{cm}$$

$$m_1 g x_1 + m_2 g x_2 = (m_1 g + m_2 g) x_{cm}$$

$$\boxed{m_1 x_1 + m_2 x_2 = (m_1 + m_2) x_{cm}}$$

1. Look at a system where you have a 100 g mass 70 cm from a 200 g mass. Where is

the centre of mass of the system?

2. What if you hang a 100g mass at 10cm on a 150g metre stick and a 200g mass at 80 cm on the metre stick. Where would you put your finger to balance the system? What force would you need to exert?

   Trick for question 2: the mass of the metre stick can be treated as a point mass at the centre (50cm).

$$m_1 r_1 = m_2 (r - r_1)$$

$$m_1 x_1 + m_2 x_2 = (m_1 + m_2) x_{cm}$$

$$100g(0) + 200g(70cm) = (300g) x_{cm}$$

$$\boxed{x_{cm} = 46 cm}$$

$$m_1 x_1 + m_2 x_2 + m_3 x_3 = (m_1 + m_2 + m_3) x_{cm}$$

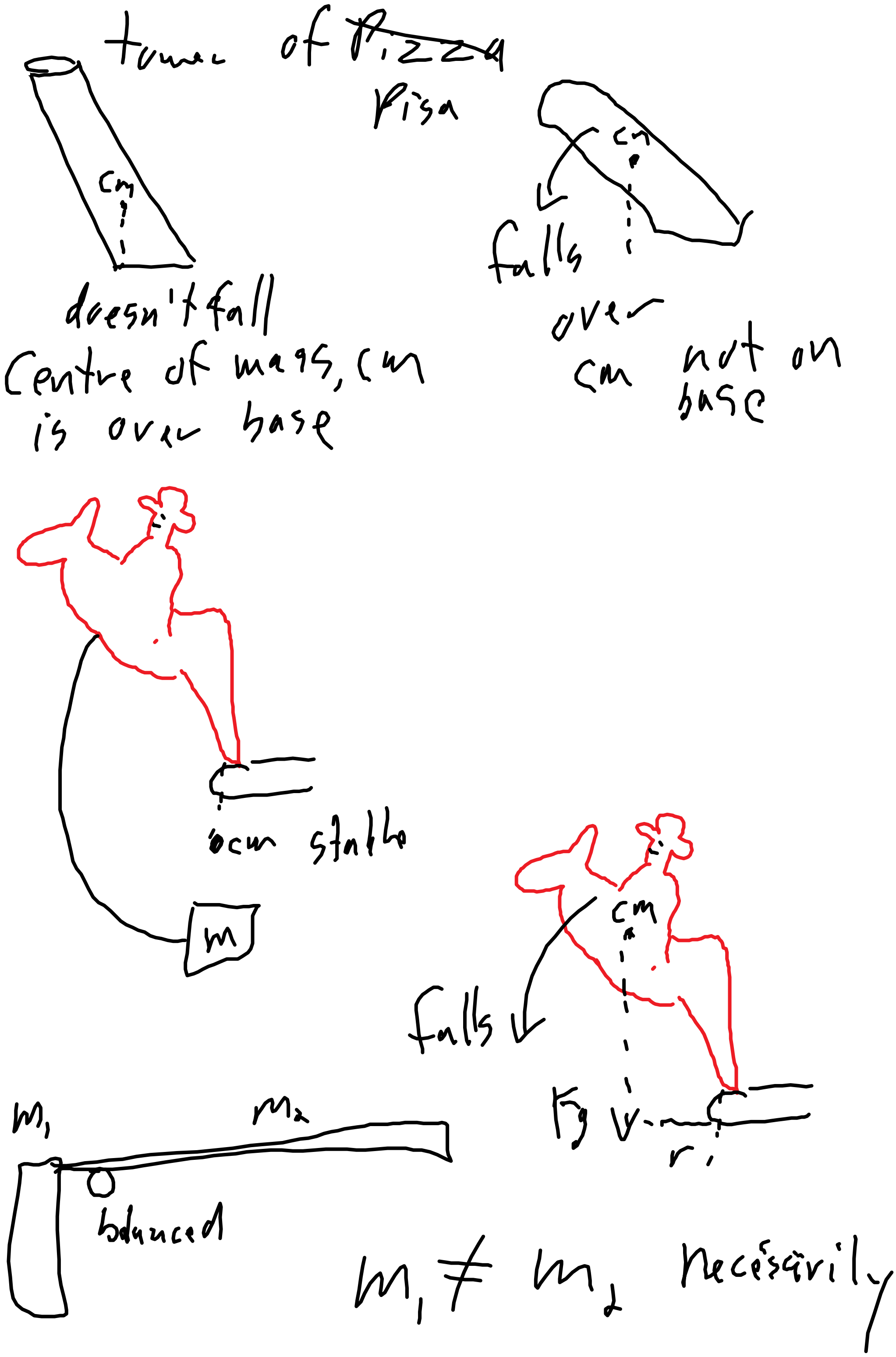
$$100g(10cm) + 200g(80cm) + 150g(50cm) = (100+200+150) x_{cm}$$

$$\boxed{x_{cm} = 54 cm}$$

Block 1-3

Centre of Mass + Statics

tower of ~~Pizza~~

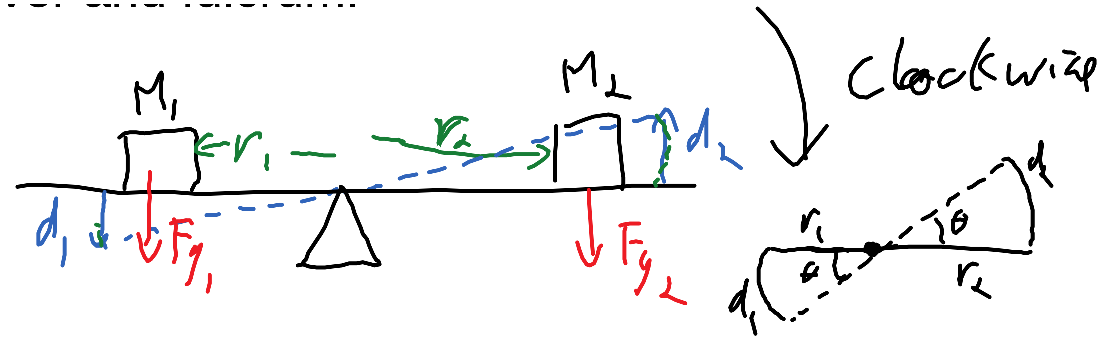


Centre of Mass is NOT the centre point of the mass, it is where the distributions of mass in space is centred.

Look at a teeter-totter (see-saw), a simple lever and fulcrum.

M1 M2) clockwise

lever and fulcrum...



Work = ΔEnergy

$W = Fd$

$W_1 = W_2$

$F_{g_1} d_1 = F_{g_2} d_2$

$d_1 \propto r_1$

$d_2 \propto r_2$

So

$F_{g_1} r_1 = F_{g_2} r_2$ ← not work - this is torque, ↑ ↑ greek letter tau

Torque, τ, is the vector cross product of force, F, and radial distance r.

(recall dot product of W=Fdcosθ)

τ= Fr sin θ

Maximum when F and r are perpendicular

Translational equilibrium - no acceleration
Net force is zero
$F_{up}=F_{down}$ $F_{right}=F_{left}$

Rotational equilibrium - the rotation is not speeding up or slowing down.
Sum of all torques = 0
Or torques clockwise = torques counter-clockwise

Static equilibrium - both translational and rotational equilibrium

look back at See-Saw

$$\Sigma \tau = 0$$

(epsilon → Σ; tau → τ; equals → $=$; zero → 0)

$$\tau_c = \tau_{cc}$$

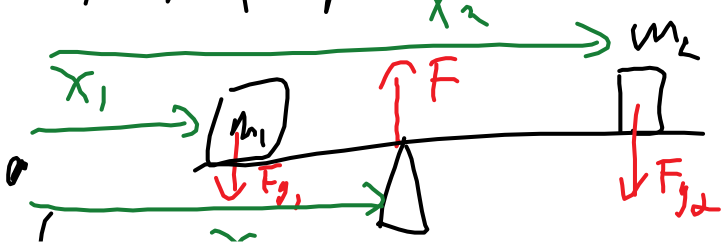
Set reference point at centre of mass

$$\tau_c = \tau_{cc}$$

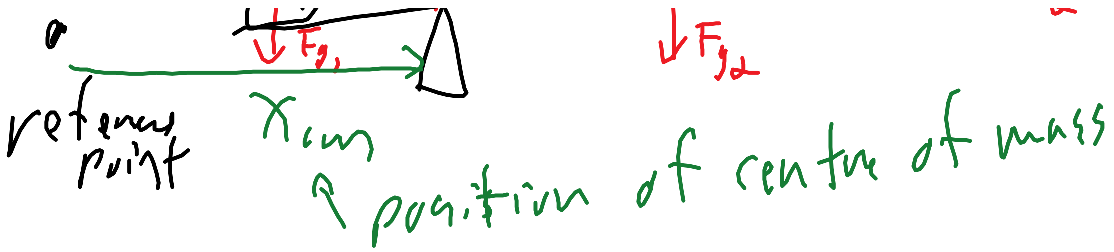
$$F_{g_2} r_2 = F_{g_1} r_1$$

(why? why not?)

Set ref point to the left



$$F = F_{g_1} + F_{g_2}$$



$$\tau_c = \tau_{cc}$$

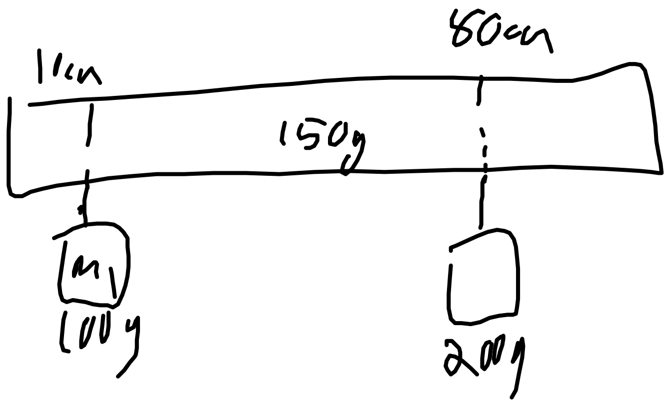
$$F_{g_1} x_1 + F_{g_2} x_2 = F x_{cm}$$

$$m_1 g x_1 + m_2 g x_2 = (m_1 g + m_2 g) x_{cm}$$

$$m_1 x_1 + m_2 x_2 = (m_1 + m_2) x_{cm}$$

Eg A 150g metre stick has a 100g mass at 10cm and a 200 g mass at 80cm. Where should you put your finger to balance?

P170-171 Q39, 40, 45, 47, 49



$$m_1 x_1 + m_2 x_2 + m_3 x_3 = (m_1 + m_2 + m_3) x_{cm}$$

$$100g(10cm) + 200g(80cm) + 150g(50cm) = (100+200+150)X_{cm}$$

$$\boxed{X_{cm} = 54\,cm}$$